MECH. RES. COMM. Vol. 4(4), 241-246, 1977. Pergamon Press. Printed in USA.

HALFSPACES

A.P.S. Selvadurai
Department of Civil Engineering, Carleton University, Ottawa, Canada

(Received 4 April 1977; accepted for print 28 May 1977)

Introduction

The present work is concerned with the axisymmetric interaction of a Winkler medium contained between two homogeneous, isotropic elastic halfspaces. The Winkler medium, which is composed of a dense array of independent spring due to the external traction $q(r)$ is given by

$$\bar{w}_q^0(\xi) = \frac{a(1-\nu)}{G\xi}\,\bar{q}^0(\xi) \tag{1}$$

where a is a typical length parameter of the problem and G and ν are respectively the linear elastic shear modulus and Poisson's ratio of the elastic material. In (1) $\bar{w}_q^0(\xi)$ denotes the zeroth-order Hankel transform [illegible]

$$\bar{w}_q^0(\xi) = H\{w_q(r);\xi\} = \int_0^\infty r w_q(r) J_0(\xi r/a)\,dr \tag{2}$$

and $\bar{q}^0(\xi)$ denotes the zeroth-order Hankel transform of the normal surface traction $q(r)$. The corresponding Hankel inversion theorem is

$$w_q(r) = \frac{1}{a^2}\int_0^\infty \xi \bar{w}_q^0(\xi) J_0(\xi r/a)\,d\xi . \tag{3}$$

By considering the solution to the Mindlin force problem [3, 4] it can be [illegible]

shown that the transformed value for the surface displacement $w_p(r)$ is given by

$$\bar{w}_p^0(\xi) = -\frac{a(1-\nu)}{G\xi}\,\bar{S}^0(\xi) \tag{4}$$

where

[illegible]

Compression of The Winkler Layer

The results described above can now be employed to obtain expressions for the transformed values of the surface displacements of the halfspaces under the combined action of the stress in the Winkler layer and the internal loading [illegible]

[illegible]

Identifying the stress in the Winkler layer as $q(x)$, the transformed expres- [illegible]

[illegible]

$$\frac{w_m(r)}{[ka^2\,(1-\nu_m)/G_m]} = \int_0^\infty \frac{\left[\dfrac{(1-\nu_n)}{\xi a^2 G_n}\{\bar{S}_n^0(\xi) - \bar{S}_m^0(\xi)\} - \bar{S}_m^0(\xi)\right]}{\left[1 + \dfrac{ka}{\xi}\{\dfrac{(1-\nu_m)}{G_m} + \dfrac{(1-\nu_n)}{G_n}\}\right]} J_0(\xi r/a)\, d\xi \,. \qquad (10)$$

The specific expressions for the surface displacements of the halfspace regions 1 and 2 are recovered by substituting m = 1, n = 2 and m = 2, n = 1 in (10), respectively. In addition, the contact stress q(r) can be directly employed to derive appropriate expressions for the stresses and displacements in the two halfspace regions.

Evaluation of The Infinite Integrals

The general numerical evaluation of the integral expressions for the surface [illegible]

direct numerical integration technique. Briefly, such numerical integration is performed by representing the integrand as an infinite series bounded by subsequent zeros of $J_0(\xi r/a)$. The application of a Gauss-Legendre quadrature technique for the evaluation of each interval of the integrand yields rapidly

$$+ \frac{(1-\nu_m)\ kP_n c_n}{4\pi G_m G_n a} [\{\frac{a}{c_n} + (\lambda-\mu_n)e^{\lambda c_n/a}\ Ei(-\lambda c_n/a)\} + \quad (12)$$

$$- \{\frac{a}{c_m} + (\lambda-\mu_m)e^{\lambda c_m/a}\ Ei(-\lambda c_m/a)\} (\frac{1-\nu_n}{1-\nu_m}\ \frac{P_m c_m}{P_n c_n})]\ .$$

In (11) and (12) $|\arg\lambda| < \pi$; $(c_i/a) > 0$ and

$$\mu_\alpha = \frac{2(1-\nu_\alpha)a}{c_\alpha} \ ; \quad \lambda = ka\ [\frac{(1-\nu_m)}{G_m} + \frac{(1-\nu_n)}{G_n}]\ ; \quad (13)$$

also Ei(-x) is the exponential integral, which is related to the function $E_1(x)$ according to $Ei(-x) = -E_1(x)$; tabulated numerical values for $E_1(x)$

case of identical loading by identical halfspaces the equations (11) and (12) reduce to the convenient forms

$$\frac{q(0)}{\ldots} = \lambda\{\mu-\lambda\}e^{\lambda}Ei(-\lambda) + \{\mu - \lambda + 1\}\ .$$

w(0)

[3] W.R. Dean, H.W. Parsons and I.N. Sneddon, Proc. Camb. Phil. Soc., 40, 5 (1944)

[4] R.D. Mindlin, Physics, 7, 195 (1936)

[5] I.N. Sneddon, G.M.L. Gladwell and S. Coen, Letters Appl. and Engng. Sci [illegible] 3, 1 (1975)

[6] V.I. Pagurova, Tables of the Exponential Integral $E_{\nu}(x) = \int^{\infty} e^{-xu} u^{-\nu} du$, [illegible]

[7] M. Abramowitz and I.A. Stegun, Handbook of Mathematical Functions, Dover, N.Y. (1965)